

LOW DENSITY INSTABILITY IN A NUCLEAR FERMI LIQUID DROP

V.M. Kolomietz^{1,2)} and S. Shlomo¹⁾

¹⁾*Institute for Nuclear Research, Prosp. Nauki 47, 252028 Kiev, Ukraine*

²⁾*Cyclotron Institute, Texas A&M University, College Station, Texas 77843*

Abstract

The instability of a Fermi-liquid drop with respect to bulk density distortions is considered. It is shown that the presence of the surface strongly reduces the growth rate of the bulk instability of the finite Fermi-liquid drop because of the anomalous dispersion term in the dispersion relation. The instability growth rate is reduced due to the Fermi surface distortions and the relaxation processes. The dependence of the bulk instability on the multipolarity of the particle density fluctuations is demonstrated for two nuclei ^{40}Ca and ^{208}Pb .

PACS number: 25.70-z, 25.70.Mn, 24.75.+i

I. INTRODUCTION

In the vicinity of the equilibrium state of the nuclear Fermi-liquid drop the stiffness coefficients are positive and the system is stable with respect to particle density and surface distortions. With decreasing bulk density or increasing internal excitation energy (temperature) the liquid drop reaches the regions of mechanical or thermodynamical instabilities with respect to small particle density and shape fluctuations and to separation into liquid and gas phases. The process of the development of instability is a complicated one. We will discuss some aspects of the process. In particular, we will study the influence of a sharp liquid-drop boundary on the instability with respect to small particle density fluctuations. In actual nuclear processes (heavy-ion reactions, nuclear fission etc.) nuclear matter is not static, and consequently the development of instability depends not only on the equation of state, but also on the dynamical effects such as the dynamical Fermi-surface distortion or the relaxation processes. We will take into account these aspects in studying the stability of the Fermi-liquid drop in both regimes of the first- and zero sound modes.

II. BULK INSTABILITY OF THE FERMI-LIQUID DROP

Let us consider small density fluctuations $\delta\rho(\mathbf{r}, t)$ starting from the nuclear fluid dynamic approach [1,2]. The linearized equation of motion reads (see Ref. [3]),

$$m \frac{\partial^2}{\partial t^2} \delta\rho = \vec{\nabla} \rho_{eq} \vec{\nabla} \frac{\delta E}{\delta\rho} + \nabla_\nu \nabla_\mu P'_{\nu\mu}, \quad (1)$$

where ρ_{eq} is the equilibrium density, E is the total energy and the pressure tensor, $P'_{\nu\mu}$, represents the deviation of the pressure from its isotropic part due to the Fermi surface distortions.

The variational derivative $\delta E/\delta\rho$ in Eq. (1) implies a linearization with respect to the density variation $\delta\rho$:

$$\frac{\delta E}{\delta\rho} = \left(\frac{\delta E}{\delta\rho} \right)_{eq} + \hat{L}[\rho_{eq}] \delta\rho + \mathcal{O}(\delta\rho)^2. \quad (2)$$

We point out that the first term on the r.h.s. of Eq. (2) does not enter Eq. (1) because of the equilibrium condition $(\delta E/\delta \rho)_{eq} = \lambda_F = \text{const}$, where λ_F is the chemical potential. The operator \hat{L} can be derived from the equation of state $E = E[\rho]$. We will use the extended Thomas-Fermi approximation for the internal kinetic energy [5] and the Skyrme-type forces for the interparticle interaction [4]. In the special case of a spin saturated and charge conjugated nucleus, and neglecting spin-orbit and Coulomb effects, the equation of state reads

$$E[\rho] = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m} \left[\frac{3}{5} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3} + \frac{1}{4} \eta \frac{(\vec{\nabla} \rho)^2}{\rho} \right] + \frac{3}{8} t_0 \rho^2 + \frac{1}{16} t_3 \rho^3 + \frac{1}{64} (9 t_1 - 5 t_2) (\vec{\nabla} \rho)^2 \right\}. \quad (3)$$

The effective forces used in Eq. (3) leads to an overestimate of the incompressibility coefficient. This is a well-known feature of Skyrme forces which can be overcome by taking non-integer powers of ρ in the potential energy density in Eq. (3). For our purposes we shall, however, be content with the form (3). To make quantitative estimates of the finite size effects on the bulk instability of the liquid drop, we will assume a sharp surface behaviour of $\rho_{eq}(\mathbf{r})$ having a bulk density ρ_0 and an equilibrium radius R_0 . Taking into account Eqs. (2) and (3), the operator $\hat{L}[\rho_{eq}]$ is then reduced to the following form

$$\hat{L}[\rho_{eq}] \delta \rho = \frac{K}{9} \nabla^2 \delta \rho - 2 (\beta + t_s \rho_0) \nabla^2 \nabla^2 \delta \rho \quad \text{at } r < R_0, \quad (4)$$

where

$$\beta = \frac{\hbar^2}{8m} \eta, \quad t_s = \frac{1}{64} (9 t_1 - 5 t_2)$$

and K is the incompressibility coefficient

$$K = 6 e_F (1 + F_0) \left(1 + \frac{1}{3} F_1 \right)^{-1}. \quad (5)$$

The Landau parameters F_l are given by

$$F_0 = \frac{9 \rho_0}{8 e_F} \left[t_0 + \frac{3}{2} t_3 \rho_0 \right] \frac{m^*}{m} + 3 \left(1 - \frac{m^*}{m} \right), \quad F_1 = 3 \left(\frac{m^*}{m} - 1 \right), \quad (6)$$

where

$$\frac{m}{m^*} = 1 + \frac{m \rho_0}{8 \hbar^2} (3 t_1 + 5 t_2)$$

and e_F is the Fermi energy.

The pressure tensor $P'_{\nu\mu}$ can be expressed through the displacement field $\vec{\chi}(\mathbf{r}, t)$ [6]. Assuming also $\delta\rho \sim e^{-i\omega t}$, the pressure tensor $P'_{\nu\mu}$ is given by [3]

$$P'_{\nu\mu} = \frac{i\omega\tau}{1 - i\omega\tau} P_{eq} \Lambda_{\nu\mu}, \quad (7)$$

where τ is the relaxation time and we used the symbol

$$\Lambda_{\nu\mu} = \nabla_\nu \chi_\mu + \nabla_\mu \chi_\nu - \frac{2}{3} \delta_{\nu\mu} \nabla_\lambda \chi_\lambda \quad (8)$$

for this combination of gradients of the Fourier transform χ_ν of the displacement field $\vec{\chi}(\mathbf{r}, t)$.

The equilibrium pressure of a Fermi gas, P_{eq} , in Eq. (7), is given by

$$P_{eq} = \frac{1}{3m} \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} p^2 f_{eq}(\mathbf{r}, \mathbf{p}) \approx \rho_0 p_F^2 / 5 m,$$

where $f_{eq}(\mathbf{r}, \mathbf{p})$ is the equilibrium phase-space distribution function and p_F is the Fermi momentum. We point out that Eq. (7) is valid for arbitrary relaxation time τ and thus describes both the zero- and the first-sound limit as well as the intermediate case.

Taking into account the continuity equation and Eqs. (4), (7) and (8), the equation of motion (1) can be reduced in the nuclear interior to the following form (we consider the isoscalar mode):

$$- m \omega^2 \delta\rho = \left(\frac{1}{9} K - \frac{4}{3} \frac{i\omega\tau}{1 - i\omega\tau} (P_{eq}/\rho_0) \right) \nabla^2 \delta\rho - 2 (\beta + t_s \rho_0) \nabla^2 \nabla^2 \delta\rho. \quad (9)$$

The solution of Eq. (9) for a fixed multipolarity L is given by

$$\delta\rho(\mathbf{r}, t) = \rho_0 j_L(qr) Y_{LM}(\theta, \phi) \alpha_{LM}(t), \quad (10)$$

where q is the wave number and $\alpha_{LM}(t)$ is the amplitude of the density oscillations. We will distinguish between stable and unstable regimes of density fluctuations. In the case of

a stable mode at $K > 0$, a solution of Eq. (9) of the form (10) has the following dispersion relation

$$\omega^2 = u^2 q^2 - i \omega \frac{\gamma(\omega)}{m} q^2 + \kappa_s q^4. \quad (11)$$

Here, u is the sound velocity

$$u^2 = u_1^2 + \kappa_v, \quad (12)$$

where u_1 is the velocity of the first sound

$$u_1^2 = \frac{1}{9m} K, \quad (13)$$

$\gamma(\omega)$ is the viscosity coefficient

$$\gamma(\omega) = \frac{4}{3} \text{Re} \left(\frac{\tau}{1 - i \omega \tau} \right) \frac{P_{eq}}{\rho_{eq}} \quad (14)$$

and

$$\kappa_v = \frac{4}{3} \text{Im} \left(\frac{\omega \tau}{1 - i \omega \tau} \right) \frac{P_{eq}}{m \rho_{eq}}, \quad \kappa_s = \frac{2}{m} (\beta + t_s \rho_0). \quad (15)$$

The quantities κ_v and $\gamma(\omega)$ appear due to the Fermi-surface distortion effect. The dispersion relation (11) determines both the real and the imaginary part of the eigenfrequency ω .

The equation of motion (9) has to be augmented by the boundary condition. This is given by a condition of the balance of the surface pressure δP_{surf} with the volume sound pressure δP_{sound} on a free surface of the liquid drop, see Refs. [7,8]. It reads

$$m u^2 \rho_0 j_L(q R_0) = \frac{1}{q^2 R_0^2} (L - 1) (L + 2) \sigma \frac{\partial j_L(qr)}{\partial r} \Big|_{r=R_0}. \quad (16)$$

Let us consider now the volume instability regime, $K < 0$, and introduce a growth rate $\Gamma = -i \omega$ (Γ is real, $\Gamma > 0$), see Ref. [9]. Using Eq. (11), one obtains

$$\Gamma^2 = |u_1|^2 q^2 - \zeta(\Gamma) q^2 - \kappa_s q^4, \quad (17)$$

where

$$\zeta(\Gamma) = \frac{4}{3m} \frac{\Gamma\tau}{1 + \Gamma\tau} \frac{P_{eq}}{\rho_0}. \quad (18)$$

Equation (17) is valid for arbitrary relaxation time τ . From it one can obtain the leading order terms in the different limits mentioned above.

(i) *Frequent collision regime: $\tau \rightarrow 0$.*

The contribution from the dynamic distortion of the Fermi surface, κ_v , can be neglected in this case and we have from Eqs. (17) and (14),

$$\Gamma^2 = |u_1|^2 q^2 - \Gamma (\tilde{\gamma}/m) q^2 - \kappa_s q^4, \quad (19)$$

where $\tilde{\gamma} = (8/15) e_F \tau$ is the viscosity coefficient. In the case of small viscosity coefficient $\tilde{\gamma}$, one has from Eq. (19)

$$\Gamma^2 \approx |u_1|^2 q^2 - \kappa_s q^4 - \frac{\tilde{\gamma}}{m} q^2 \sqrt{|u_1|^2 q^2 - \kappa_s q^4}. \quad (20)$$

The amplitude of the density oscillations, $\delta\rho_L(\mathbf{r}, t)$, grows exponentially if $\Gamma > 0$. Expression (20) determines two characteristic values of the wave number q , namely, q_{max} where the growth rate reaches a maximum of Γ_{max} , and q_{crit} where Γ goes to zero, i.e., (see also [10]),

$$\Gamma = \Gamma_{max} \text{ at } q = q_{max} < q_{crit}, \text{ and } \Gamma = 0 \text{ at } q = q_{crit}. \quad (21)$$

The values of q_{max} and q_{crit} are obtained from, see Eq. (19),

$$\left. \frac{\partial \Gamma}{\partial q} \right|_{q=q_{max}} = 0 \quad \text{and} \quad q_{crit}^2 = \frac{|u_1|^2}{\kappa_s}, \text{ at } u_1^2 < 0. \quad (22)$$

Thus, the critical wave number q_{crit} does not depend on the viscosity. However, the presence of viscosity reduces the instability, see also Fig. 1 below.

(ii) *Rare collision regime: $\tau \rightarrow \infty$.*

In this case, we have from Eqs. (17), (15) and (14)

$$\Gamma^2 = |u_1|^2 q^2 - \kappa'_v q^2 - \kappa_s q^4, \quad (23)$$

where

$$\kappa'_v = \frac{4}{3m} \frac{P_{eq}}{\rho_{eq}}. \quad (24)$$

The critical value q_{crit} and the value q_{max} are given by

$$q_{crit}^2 = \frac{|u_1|^2 - \kappa'_v}{\kappa_s}, \quad q_{max}^2 = \frac{1}{2} q_{crit}^2. \quad (25)$$

Thus, the distortion of the Fermi-surface leads to a decrease of the critical value q_{crit} , i.e., the Fermi-liquid drop becomes more stable with respect to the volume density fluctuations due to the dynamic Fermi-surface distortion effects.

III. NUMERICAL RESULTS AND DISCUSSION

In Fig. 1 we have plotted the instability growth rate Γ as obtained from Eq. (17). The calculation was performed for the Skyrme force SIII. The relaxation time was taken in the form $\tau = \hbar \alpha / T^2$ [11] with $\alpha = 9.2 \text{ MeV}$ and $\alpha = 2.6 \text{ MeV}$ [12] and the bulk density ρ_0 was taken as $\rho_0 = 0.3 \rho_{sat}$, where ρ_{sat} is the saturated density $\rho_{sat} = 0.1453 \text{ fm}^{-3}$. We show also the result for the nonviscous infinite nuclear matter and the nonviscous finite liquid drop neglecting Fermi surface distortion effects. In a finite system, the non-monotony behaviour of the instability growth rate as a function of the wave number q is due to the anomalous dispersion term in Eq. (11) created by the gradient terms in the equation of state. We point out that the finite system becomes more stable with respect to short-wave-length density fluctuations at $q > q_{max}$. We can also see that the presence of viscosity decreases the instability. The strong decrease of instability in a Fermi liquid drop (FLD), when compared with the corresponding result for the usual liquid drop (LD), is because of the Fermi surface distortion effects. In Fig. 2, this peculiarity of the FLD can be seen in a transparent way for both the infinite nuclear matter and the finite Fermi liquid drop.

For a saturated nuclear liquid one has for the force parameters $t_0 < 0$, $t_3 > 0$ and $t_s > 0$. Thus, the critical value q_{crit} , Eq. (22), increases with decreasing bulk density ρ_0 at $u_1^2 < 0$, see also Eq. (6). The existence of the critical wave number q_{crit} for an unstable mode is a feature of the finite system. The growth rate Γ depends on the multipolarity L of the nuclear

density distortion and on the position of the eigenvalue, q_L , in the interval of $q = 0 \div q_{crit}$ [10]. For a given R_0 , the value of q_L increases with L for $L \geq 2$ because of the boundary condition (16), see Table 1. That means that if $q_L < q_{max}$ the instability increases with L and the nucleus becomes more unstable with respect to an internal clusterization to small pieces (high multipolarity regime) rather than to binary fission (low multipolarity regime). In contrast, the binary fission is preferable if $q_{max} < q_L < q_{crit}$.

We give in Table 1 the values q_L/k_F for two nuclei, ^{208}Pb and ^{40}Ca as obtained from Eq. (16). The calculations were performed with the surface tension parameter $4\pi r_0^2\sigma = 17.2 \text{ MeV}$. We point out that the value of q_{max} is given here by $q_{max}/k_F = 0.69$. In Fig. 3 we have plotted the instability growth rate at $T = 6 \text{ MeV}$ and $\alpha = 9.2 \text{ MeV}$ as function of the multipolarity L of the particle density fluctuations for two nuclei ^{208}Pb and ^{40}Ca . As is seen from Fig. 3, the lowest values of $L \leq 3$ give the contribution to the instability growth rate Γ for the nucleus ^{40}Ca . Thus, the nucleus ^{40}Ca is unstable with respect to the fission under the conditions considered above. In contrast, the instability growth rate of the nucleus ^{208}Pb includes the higher multipolarity $L \leq 8$ and this nucleus has to be unstable with respect to multifragmentation.

IV. SUMMARY AND CONCLUSION

Starting from the fluid dynamic equation of motion for the Fermi liquid drop with a sharp surface, we have derived the dispersion relations (11) and (17) for both the stable and the unstable regime. The dispersion relations are influenced strongly by the Fermi-surface distortion effect and the anomalous dispersion caused by the finiteness of the system. The presence of the Fermi surface distortion enhances the stiffness coefficient for a stable mode and reduces the instability growth rate for an unstable one.

We have shown that the instability growth rate in an unstable finite system is a non-monotony function of the wave number q because of the anomalous dispersion term. This is in contrast with the infinite nuclear matter case where the instability growth rate increases

with q . The non-monotony behaviour of the instability growth rate $\Gamma(q)$ in a finite Fermi liquid drop is accompanied with two characteristic wave numbers q_{max} and q_{crit} , see Eqs. (21) and (22). The distortion of the Fermi-surface leads to a decrease of the critical value q_{crit} . The decay mode of an unstable Fermi liquid drop depends on the location of the eigen wave number q on the slope of the curve $\Gamma(q)$. The Fermi liquid drop is more unstable with respect to multifragmentation if $q < q_{max}$ and the binary fission is preferable if $q > q_{max}$. This is because the eigen wave number q_L , derived from the secular equation (16), increases with the multipolarity L of the particle density fluctuations. As an example, we have demonstrated this phenomenon in the case of hot nuclei ^{40}Ca and ^{208}Pb . The nucleus ^{40}Ca is more unstable with respect to the short wave fluctuations and prefers to decay into the binary fission channel. The multifragmentation channel is preferable at the development of the instability in the heavy nucleus ^{208}Pb .

V. ACKNOWLEDGEMENTS

This work was supported in part by the US Department of Energy under grant # DOE-FG03-93ER40773 and the INTAS under grant # 93-0151. We are grateful for this financial support. One of us (V.M.K.) thanks the Cyclotron Institute at Texas A&M University for the kind hospitality.

REFERENCES

- [1] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, (Springer-Verlag, New York, 1980), Ch. 13.
- [2] V.M. Kolomietz, *Local Density Approach for Atomic and Nuclear Physics* (Naukova Dumka, Kiev, 1990) (in Russian).
- [3] D. Kiderlen, V.M. Kolomietz and S. Shlomo, Nucl.Phys. **A608**, 32 (1996).
- [4] Y.M. Engel, D.M. Brink, K. Goeke, S.J. Krieger and D. Vautherin, Nucl. Phys. **A249** (1975) 215.
- [5] R.K. Bhaduri and C.K. Ross, Phys. Rev. Lett. **27** (1971) 606.
- [6] V.M.Kolomietz, A.G.Magner and V.A.Plujko, Z. Phys. **A345**, 131, 137 (1993).
- [7] H.Lamb, *Hydrodynamics* (Dover, New York, 1945).
- [8] A. Bohr and B.R. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975).
- [9] C.J.Pethick and D.G.Ravenhall, Ann. of Phys. **183**, 131 (1988).
- [10] C.J.Pethick and D.G.Ravenhall, Nucl. Phys. **A471**, 19c (1987).
- [11] A.A. Abrikosov and I.M. Khalatnikov, Rep. Prog. Phys. **22**, 329 (1959).
- [12] V.M.Kolomietz, V.A.Plujko and S.Shlomo, Phys. Rev. C **52**, 2480 (1995).
- [13] J.R. Nix and A.J. Sierk, Phys. Rev. **C21**, 396 (1980).

TABLES

TABLE I. Values of q_L/k_F , as obtained from Eq. (16) for the surface tension parameter σ derived from $4\pi r_0^2\sigma = 17.2\text{ MeV}$ and $T = 1\text{ MeV}$, for two nuclei ^{208}Pb and ^{40}Ca .

L	2	3	4	5	6	7	8
^{40}Ca	0.986	1.225	1.457	1.684	1.907	2.128	2.347
^{208}Pb	0.569	0.707	0.841	0.972	1.101	1.228	1.355

FIGURES

FIG. 1. The dependence of the instability growth rate Γ on the wave number q . The calculations were performed for Skyrme force SIII, temperature $T = 6 \text{ MeV}$ and density $\rho_0 = x \rho_{sat}$ with $x = 0.3$. The solid curves are for the Fermi liquid drop from Eq. (17); the values $\alpha = 9.2 \text{ MeV}$ and $\alpha = 2.6 \text{ MeV}$ of the relaxation time parameters are shown as labels to the curves. The dashed lines are the results for the nonviscous liquid without the Fermi surface distortion effects: curve (1) is the result for an infinite matter and curve (2) is for a finite liquid drop.

FIG. 2. The dependence of the instability growth rate Γ on the wave number q for the infinite matter (dashed lines) and the finite system. The curves FLD and (2) are for the Fermi liquid; the curves LD and (1) are for the usual liquid, i.e., neglecting the Fermi surface distortion effects. All calculations were performed with $\alpha = 0$ and $T = 1 \text{ MeV}$ and with the force parameters and ρ_0 as in Fig. 1.

FIG. 3. The dependence of the instability growth rate Γ on the multipolarity L of the particle density fluctuations for two nuclei ^{208}Pb and ^{40}Ca . The calculations were performed using the FLD results of Fig. 1 at $\alpha = 9.2 \text{ MeV}$ and $T = 1 \text{ MeV}$ and the surface tension parameter σ derived as in Table 1.

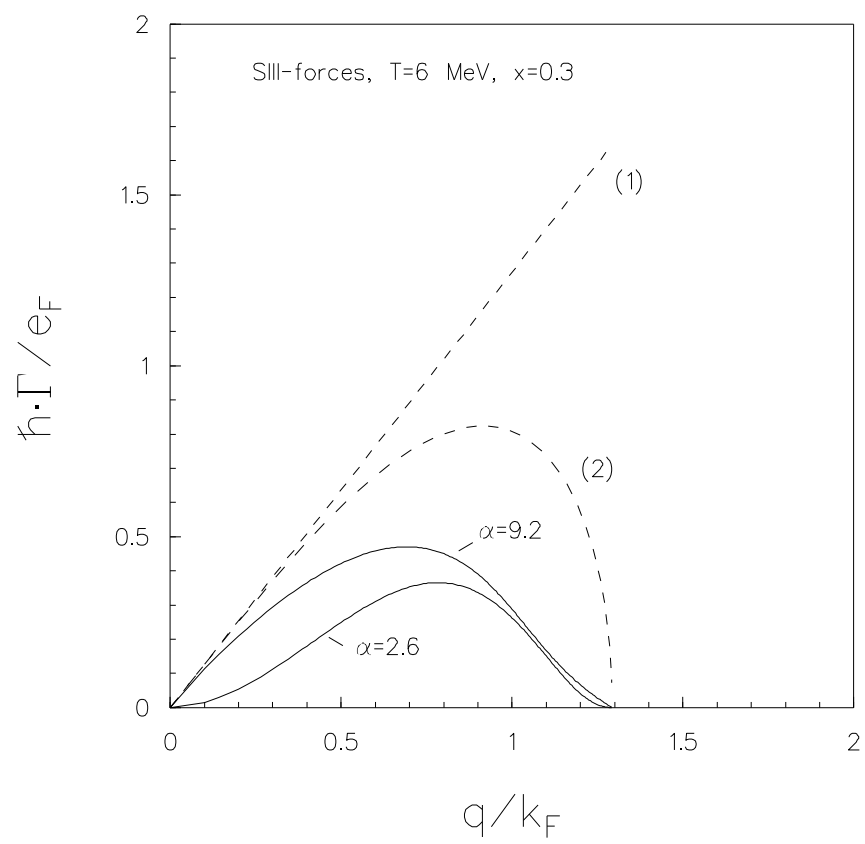


Fig.1

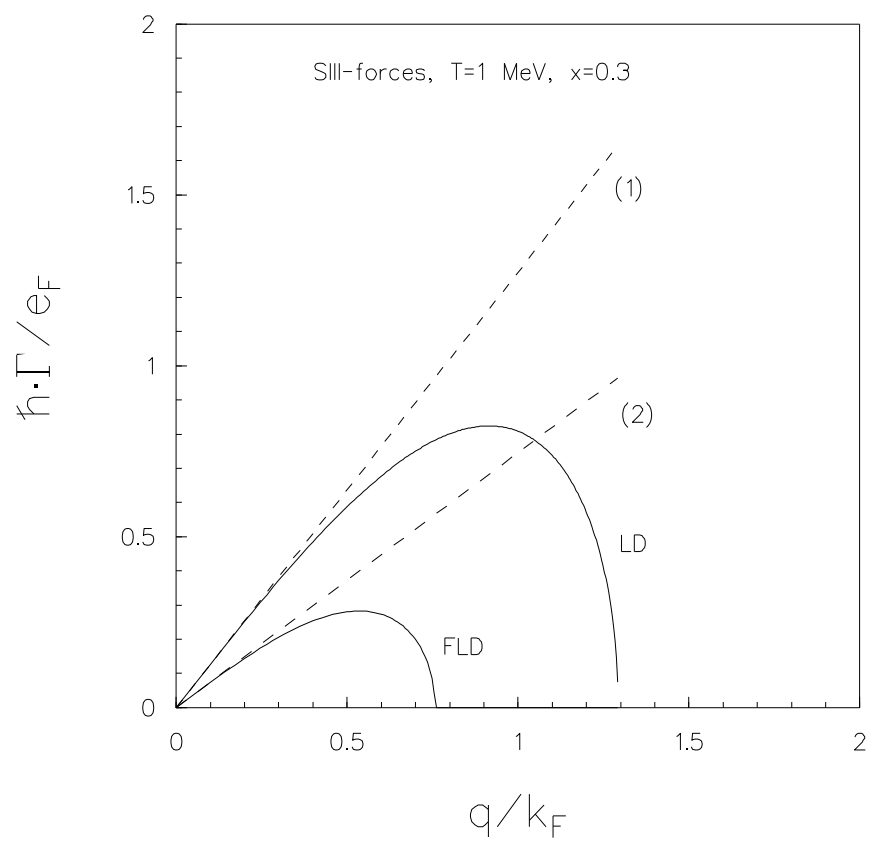


Fig.2

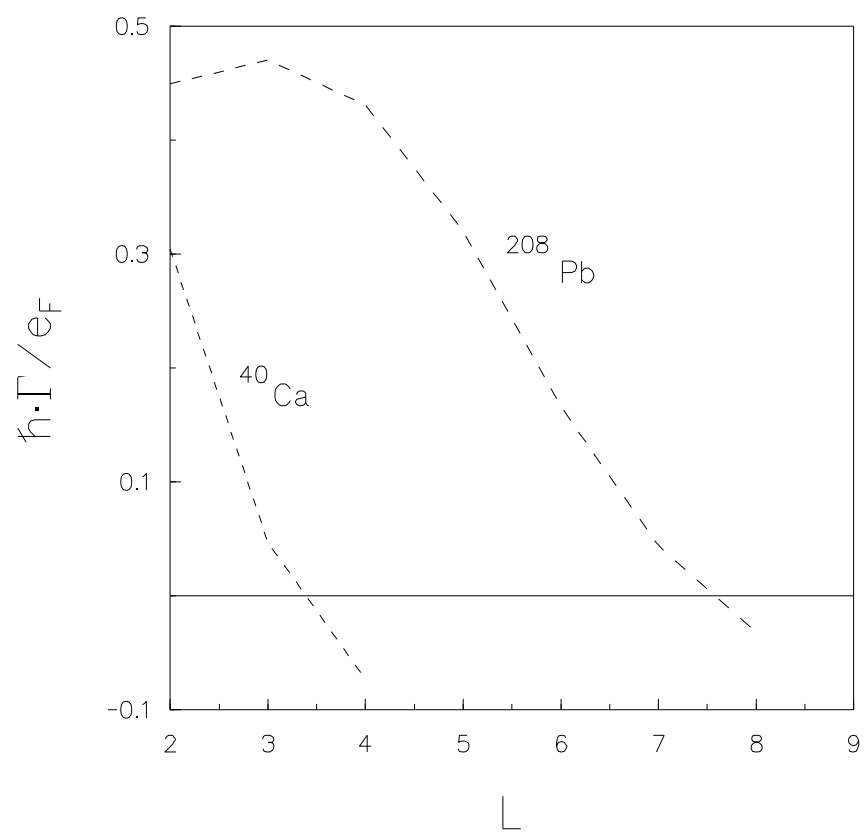


Fig.3